Chapter Nine
Making Arithmetic Meaningful

What does arithmetic mean to a child? Is arithmetic a series of rules that must be committed to memory? Is it fear of not understanding? Are apprehension and shame caused by fear of impending failure? Are a child's feelings about arithmetic related to the processes he is supposed to be learning, or are his feelings related to the judgments he will receive for lack of success in coming up with "right" answers? What are the child's intellectual and inner reactions to arithmetic? Discussions with children having difficulty with arithmetic show that they are preoccupied by rules, principles, memorization of facts, absolutes, and right and wrong. Their feelings include boredom, fear, confusion, frustration, and anger. Unfortunately, these feelings when dealing with the arithmetic process are retained throughout life.

Children's comments about their reactions to arithmetic frequently vary according to how the subject is presented and the attitude of the teacher. When a child is comfortable in the learning situation, he becomes receptive and responsive; otherwise, he will close off and his learning will be limited, distorted, or nonexistent.

Arithmetic learning should be a product of understanding, leading to the ability to innovate, create, communicate, and use the data presented. In this chapter we attempt to develop ways of helping a child reach the goal of *production* from *understanding.*

What Was

In the past, arithmetic was usually taught by having the child do certain operations known as addition, subtraction, multiplication, and division. These operations were the result of putting numerals down on paper and doing things with them. In time, the numerals assumed the status of the real thing. They became an end in themselves. As children advanced in the educational process, they were introduced to "word problems," formulae, geometry, and other so-called reasoning processes which were based on the assumption that the children had become facile with the simpler operations. Far too often children were pushed into using higher-order processes before they had mastered the lower ones. They did not have an adequate foundation of arithmetic, and they had not developed security within themselves when dealing with numbers and numerals.

"New Math" was going to solve the problems of the child having difficulty with arithmetic by making him aware of the inner meaning of what he was doing. Unfortunately, old philosophies and methods were brought to the teaching of new concepts and they were not compatible. The result was even more confusion to teachers, children, and parents.

The language aspects of arithmetic

Arithmetic has not often, at least by laymen, been considered a language. The teaching of arithmetic, however, is enhanced when it is thought of in terms of language. Arithmetic is a language because it uses written and spoken vehicles. These symbols must be received and translated within a person's inner language. When a person tries to learn a new language he can do so only when he feels free within himself to receive the vocabulary of that language and relate it in a meaningful way to some motor and visual experience. To master the new language, it is necessary for the person to think in terms of the new language and express his thoughts in that language.

Inner language is a thought process in which a person takes information from his external world, matches and adds to it from his internal storehouse of experiences, information, and knowledge and draws appropriate conclusions. Internally a person visualizes and reconstructs situations that are described outside of himself. He restates what he hears in terms that he can understand, using visual, language, and motor mechanisms. The individual must give mathematical symbols (2, +, =) meaning internally. This meaning will be in terms of a sense of muscle movement patterns, a feeling of size, distance, direction, position, and movement. To accomplish that, a person must be receptive to information. Then he can be free to think.

Causes of failure

One of the major causes of failure in arithmetic is the closing off of inner language function by the child because of fear. Arithmetic is usually treated as an absolute: you are either right or wrong. The child sees himself as either right or wrong—usually wrong—when he "does his math."

One of my young patients told me how disappointed she is with herself whenever she works with arithmetic. She dreads the thought of doing any work involving arithmetic. This young lady makes a negative self-judgment by calling herself stupid; she then loses receptivity. Inner confusion results and she turns off. When she is in that state of mind, she cannot be taught.

Anger, fear, and negative self-judgment place the child in a position where arithmetic becomes a burdensome task because it damages the self-image. Watch a child who has trouble with arithmetic. As you try to teach him, watch his eyes glaze over. Tension and fear cause his face to harden. His mouth tightens as he talks, and his body becomes tense when you attempt to show him how "easy" and what "fun" arithmetic is. How can the child be receptive to learning when his inner language system is closed down? He turns off his thought processes to protect himself. His failures in arithmetic have already made him feel inadequate enough. Why should he expose himself to more?

The teacher must be aware of how every child feels about himself and must be cautious not to add to any negative feeling generated by the child. Effective teaching comes from listening to a child's thinking and how he expresses his thought processes. Asking a child, "How did you arrive at that answer?" is a greater stimulus to learning than "That's right!" or "That's wrong!" It is essential to keep the child open and receptive.

In the procedures that follow we try to keep a child open and receptive to learning. No principle of arithmetic is so important that it is worth learning at the price of damaging a child. The methods of teaching which are illustrated are designed to boost the child's feelings about himself as well as to help him discover the basic arithmetic concepts with which we are concerned.

Numerals and Numbers

A child looking at the words in a book is expected to translate the letters into identifiable sounds that will result in meaning. The letters of the alphabet, the written symbols of our language, are accepted as the basis for reading and written communication. Numerals, the written symbols of arithmetic, are the basis for the written and reading communication of mathematics. For numerals to have any worthwhile significance, a child must translate each numeral into a meaning beyond the configuration of that symbol. Too often teachers, parents, and children become concerned merely with the operations that can be done with numerals without considering the understanding behind those operations. Consequently, children may be able to manipulate numerals, but may at the same time have great difficulty gaining insights into the processes of arithmetic. For arithmetic to take on full and constructive meaning, children must be taught the relationships among the symbol (numeral), the meaning (number), and the mathematical processes.

When children work with arithmetic and its processes, they must speak and think, and picture and manipulate objects in relationship to their activity. Using visual, spatial, auditory, or tactile means of illustrating what they are doing in arithmetic operations adds to meaning and understanding. In addition to a verbal or written answer to an arithmetic problem, the child must also be able to state his answer in other terms, preferably in terms that can be seen by means of demonstrations with concrete objects.

Cuisenaire Rods, Stern Blocks, abacus, adding machine, clothespins, coins, and buttons are some of the concrete media that make arithmetic more than just an intellectual experience to the child.

Definition of number and numeral

Number is real. Numeral is the symbol used in place of the actual quantity. Numerals are the "words" of the symbolic language used to make statements about quantities: $2 + 2 = 4$ is a typical sentence in this symbolic language. Number can be said to exist within a person as part of his inner language system.

When Charlie sees "2," he should experience "twoness"—the idea of two. His experience is internal. If Charlie wants to vocalize what he sees, he re-translates "twoness" into the word two. Or he writes the symbol "2" or one of the other symbols commonly used for "twoness," such as "II."

The relationship between numeral and number can be made more meaningful to children by having them talk about their names. Explain that we give people names such as George, Frank, Sally, etc., in order to identify them. Just as a numeral identifies number, the child's name identifies him. (I am sure that some bright child will pop up with the fact that we are now back to giving people numerals rather than names, in order to identify them. Don't get sidetracked on that one.)

Then proceed as follows:

Ask each child to call off his first name.

List their names on the chalkboard. (Emphasize that a name may ***represent*** a person or thing, but a name is ***not*** that person or thing. In the same way, a numeral is the symbol used to name or represent a number or group of things, but a numeral is not the actual thing.)

The dialogue goes something like the following:

Teacher: As you find your name in this list, look at it and tell me if your name looks anything like you?

Children: No!

Teacher: I agree. You are better looking. What is the difference between your name and you?

Children: Our names are written; we are real. Our names are used in place of us.

Teacher: Yes, a name is a symbol which represents or is used in place of someone or something. It is not that someone or something. We use names to allow us either to write or to talk about different people or things. What is another reason we give people different names?

Children: So that we can tell them apart, one from the other.

Teacher: Yes. In arithmetic we also use different names to talk about things and tell them apart. What do we call the names for numbers in arithmetic?

Children: Numerals are used in arithmetic to name numbers.

Qualification of numeral

A child works with numerals solely for convenience, and he must realize that he is dealing with symbols, not the real thing. A numeral is meaningless if it is isolated from some object(s) – it is then a counting number. Were you to say, "I have three," you would be describing only a numeral. I would have to ask you, "What do you have three of?" You must qualify the three in order to make it meaningful. Did you have three dollars, three candy bars, three houses, or three of what? Too often children are so caught up with work sheets and drills that require working with numerals to practice arithmetic that they fail to realize that what they are doing can be represented with something meaningful. They become so involved with symbols that only the symbols are important. Children will be able to function more freely in the processes of arithmetic when they can think in terms of a numeral as a symbolic relationship between number and thing. An example of teaching a child the concept of qualifying a numeral can be done using the following dialogue as a continuation of the similarity between naming people and naming numbers:

Teacher: What do we do when you and another child have the same name?

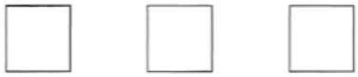
Children: We look to see if their last names are different.

Teacher: In other words, we have to use two names in order to tell who you are. One name tells us about you personally and the other name tells what family you come from. Go around the room and tell your name and what family you come from.

Children: My name is . . . . and my family name is. . . .

Teacher: The same idea works with numeral and number.

When we see



we give the number a name of three. Does the name or numeral "3" tell us very much about what we see?

Children: It tells you that you see three.

Teacher: Does the numeral "3" tell us what we see three of? Isn't it like saying, "Edward come here," when there are two Edwards in the room? How would you know to which Edward I am referring?"

Children: You would have to use Edward's last name.

Teacher: Yes, just as I would have to use Edward's last name to identify him fully, we would have to tell three of what we are talking about. We would have to say, "I see the three blocks or three squares."

This type of dialogue should be continued using examples of objects with which the children are familiar. In the classroom setting you can work with windows, doors, desks, pencils, erasers, paper clips, and any other familiar objects. The most important thing is to have the children survey their surroundings and discuss them in terms of quantity, relating number to numeral. Use the chalkboard to write the numeral and qualification of the objects the children describe to you.

This may be done as follows:

1. Should the children tell you that they see four desks, write on the chalkboard, "4 desks" and "four desks."
2. To avoid hang-ups over spelling and writing, it is extremely important that you write what the children tell you, rather than having them write it. After the children seem to get the idea behind what you are doing and relate the information freely to you, have them express what they are saying with numerals and rods. This can be done as follows, using Cuisenaire Rods, plastic numerals, word cards, and rubber stamps:

Teacher: Pointing to four chairs, ask, "How many chairs are here?"

Children: Four.

Teacher: Pick out four rods (use unit rods only) to show the number of chairs.

Children pick out four rods and place them on the desk in front of them.

On the chalkboard write, "4 chairs." Pointing to the "4," say, "Pick out the plastic numeral (or card with a numeral on it) that looks like this."

Children pick out the plastic '4'and place it on the desk in front of them.

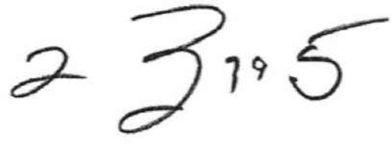
(Do a quick check for accuracy. If all children have the correct numeral, just say, "Yes" or "That's it" and go on.)

1. Should a child not have the correct numeral, say, "Does your numeral look like mine?" Have the child find the correct numeral before you go on. You could ask the child's desk mate if he agrees. *Do not* tell the child *he is wrong*. Have him see the difference under your guidance.
2. When all of the children have the correct symbol, have them pick out the card that names the same *thing* as your word. In this case have them look for the card that says, "chairs."
3. After the children pick out the correct plastic numeral, have them pick out the correct rubber stamp representing that numeral. Have them stamp the numeral on a piece of paper and trace around the stamped numeral with a crayon. The purpose of the tracing is to have the child get a feeling for the way the numeral is made.

There are many variations of ways to have a child relate number to numeral. The sequence is to have him talk about number, represent number with Cuisenaire Rods, make the symbol, see the symbol, select the symbol, and feel the symbol. To reinforce the learning, have the child then relate it back to the number by reversing the process, as follows:

- Tracing the numeral, he is to say, "This means four."
- Pointing to the plastic numeral, he is to say, "This means four."
- Pointing to the four rods, he is to say, "These are four."
- Pointing to four chairs, he is to say, "These are four."

The following illustrates the fact that numerals have meaning beyond their appearance: Copy the figures accurately in large scale.



Ask the children, "Which numeral is biggest?" Listen to the responses before you comment. Some children will tell you that the seven is biggest because it means more. Another child might tell you that the three is biggest because it looks larger. Some children may want to know if you are asking about the size of the numeral or what it represents. They will want you to qualify what you mean. The child who asks for qualification is thinking in terms of number; the child who is only looking at the size of the numeral is thinking in terms of numeral and does not remember or understand that numerals have meaning beyond their physical appearance. Explain to the children that while the "3" shown here is the largest numeral physically, the "7" represents the largest quantity of something. Another example follows:

In the numeral "56," which is greater, the "5" or the "6"?

In order to understand this question, one must know how our number system works. The "5" is greater because it means or represents 50 ones, while the "6" represents only 6 ones. If one does not understand our number system, then the "6" is greater because it follows "5" in the counting order.

Number constancy

No matter what symbol you use to represent a number—a numeral, a word, or any other designation—the number is always the same. Many times children think that there is only one way of arriving at an answer to a problem or only one way to state a number. They must gain flexibility in their understanding of ways of stating number. To illustrate this concept, have the children do the following:

Take the name of one of the children in the room and see how many nicknames they can think of for that child. Jonathan becomes Jon, Johnnie, and Jo. All of the nicknames stand for Jonathan, and regardless of which form of Jonathan is used, each still describes the same child.

The same applies to other names, such as Elizabeth, which may be represented by Betty, Liz, Beth, Eliza. Regardless of what name is used, the girl whose name is Elizabeth does not change; she still remains the same person.

The concept of constancy can be applied to number as well as names by asking the children how many different ways they can name a number. This can be done as follows:

Have the children select a number and then name it in as many ways as possible. As you do this with children, in addition to demonstrating the constancy of number, you may find out something about the child's level of arithmetic sophistication. Ask the children, "How many names can we give for this many?" (Hold up seven fingers.) List their responses on the chalkboard. You can name seven in many ways, some of which follow:

7 seven

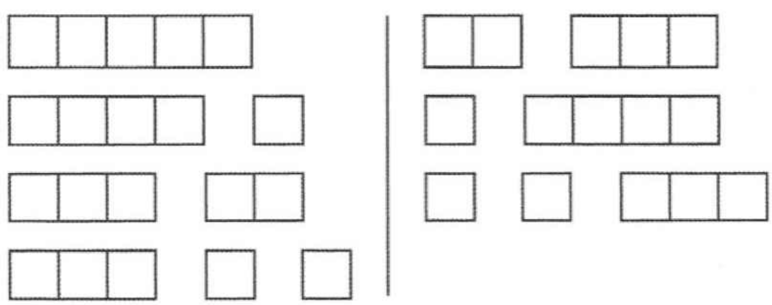
sieben (or any other language)

6 + 1	2 x 3 + 1
5 + 2	8 – 1
4 + 3	9 – 2
3 + 4	21/3
2 + 5	28/4
1 + 6	

Continue, depending on how many arithmetic processes the children want to illustrate. The illustration tells us that the person who gave those answers knows something about addition, subtraction, multiplication, division and at least one foreign language. Another way the child might demonstrate meaning or expression is through the use of physical objects, such as Cuisenaire Rods or Stem Blocks. This can be done as follows:

1. Use the unit blocks from the Cuisenaire Rods and arrange them according to number.
2. Write on the chalkboard the numeral representing that number.
3. Show that the rods may be arranged in many different ways, yet the given numeral always remains the same.
4. Use symbols to keep track of the grouping, as follows:

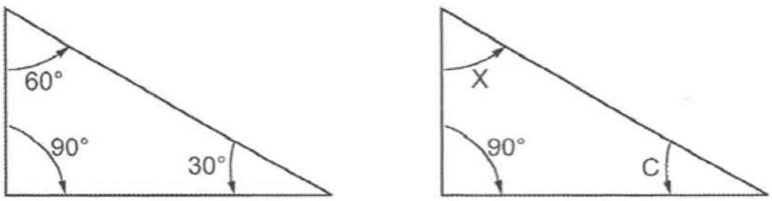
   All groupings that name five:



5. Repeat this procedure using other numbers.

Try as many ways as you know to have the children think in terms of the inner meaning of numerals. In that way children will look at arithmetic as something that is worthwhile and fun to work with because it challenges their thinking. As they become involved in higher mathematics and more advanced processes, they will look for the meaning and the understanding of what they are doing, rather than looking only for correct answers.

Lynn was very frightened of arithmetic in any form because she felt that if she did not get the right answer she would be wrong. As a result she always worked for the answer and never tried to understand what she was doing. When she was in tenth grade, she had considerable difficulty with simple algebra because she never looked for relationships, nor could she restate a problem. For example, when working with complementary angles of a right triangle, where the formula is $x + C = 90°$, and Lynn was given angle x, she could always find angle C, but when she was given angle C, she could not find angle x. This happened because she did not understand that the formula $x + C = 90°$ could be restated to read $90° - C = x$. Her failure reinforced her poor self-concept in mathematics. Her fear caused by previous failure prevented receptive, inner and expressive areas of vision and language from functioning.



Our Number System

Having an understanding and developing facility with our number system has been helpful to many children who have difficulty with arithmetic. When the child learns to understand and is conversant with the various functions of our number system, the processes of addition, subtraction, multiplication and division become less of an obstacle for him.

These understandings also help a child with our money system because both systems are based on the power of ten. We build up to ten using ones; then we build up to one hundred as a product of tens; ten hundreds become a thousand; ten thousands become one hundred thousand; and ten hundred thousands become one million. It helps a child's learning to understand how these various combinations come about and how to break them apart.

Teaching an understanding of the number system

The child must read, say (talk out), manipulate concrete material, write the symbol, verbally confirm what he has written, and compare what he has done with what was presented (confirm, monitor, or feedback). This is done in the following way, using Cuisenaire Rods:

Teaching the meaning of one

Teacher: Pick out the smallest rod from the box.

The child is asked to give this rod a name and this is done by questioning, as follows:

Teacher: Place the red (two) rod in front of the child; ask, "Can you put white rods on the red rod to make the same size as the red rod?"

Child: Yes.

Teacher: Show me.

Child: Places two white (one) rods on the red rod.

Teacher: Repeat with the other rods going one step higher each time. Then work back to the two rod. After the child has shown that he can break the red rod into two white rods, ask the child, "Can you make the white rod smaller?"

Child: No.

Teacher: What name do you think we can give that white rod?

Child: The one rod.

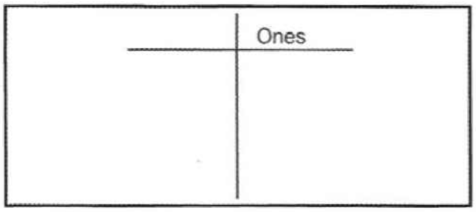
Teacher: That is the one or unit rod because it is the (pause) what?

Child: Smallest.

Teacher: Yes, it is the smallest so we call it the one rod.

Let's play with the one rod.

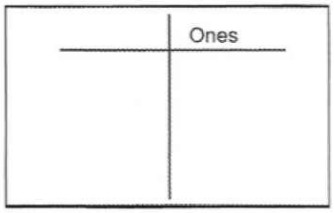
a. Draw the following on the chalkboard



b. On a sheet of 8 1/2 x 11 paper, rule off the following

c. Place the paper in front of the child.

d. Teacher: Draw a single one rod in the ones column on the chalkboard.



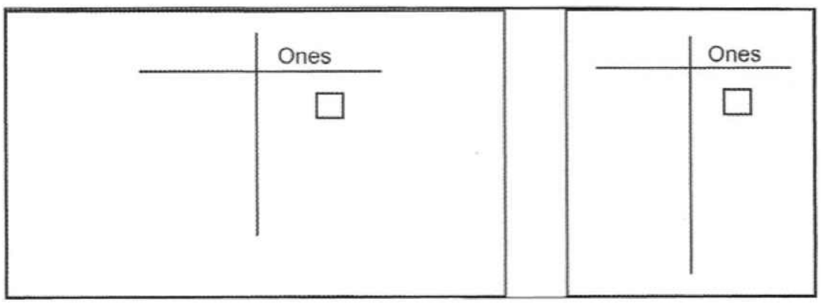
Place the same number of one rods in the ones column on your paper as you see on the chalkboard.

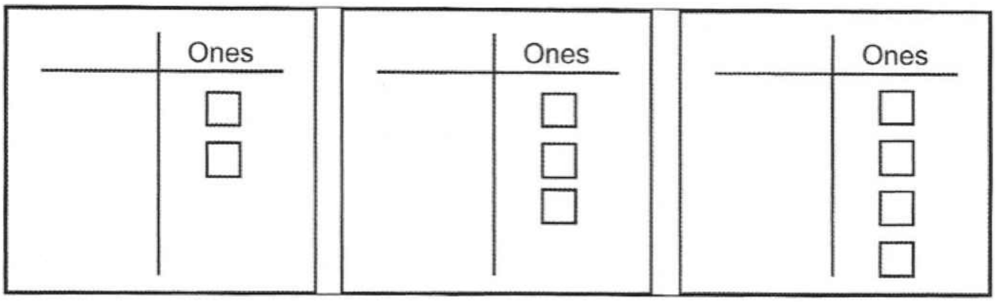
e. Teacher: How many rods do you have in the ones column?

Child: I have one rod in the ones column.

Teacher: Yes, you have one rod in the ones column and that is what I have shown to you on the chalk board.

f. Repeat above up to nine going through the same verbalizations.





Transferring from number to numeral

Eliciting the method from the child is done as follows:

Teacher: I find it difficult to draw rods on the chalkboard in order to tell you how many rods you should place in the ones column. Can someone tell me a way to do this so that I don't have to write so much?

Child: No, tell us.

Teacher: I don't understand, how can I do that?

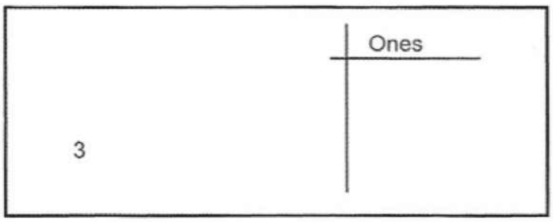
Child: Say, three rods, four rods, like that.

Teacher: How can I write it?

Child: Just write the numeral.

Teacher: Let's try what you suggest.

Teacher writes a numeral on the chalkboard as follows:



Pointing to the numeral, the teacher asks, "What is this numeral's name?"

Child: Three.

Teacher: What does this numeral say?

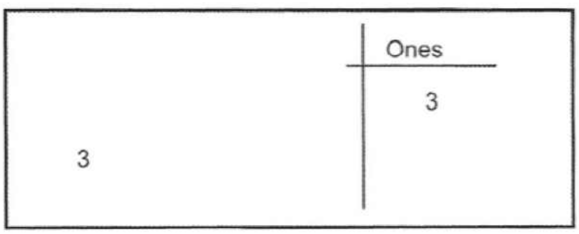
Child: Three ones.

Teacher: What does this numeral mean?

Child: Three ones.

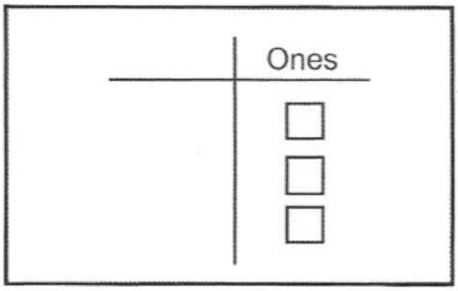
Teacher: Where should I write it?

Child: In the ones column.



Teacher: What are you going to do?

Child: Put three rods in the ones column on my paper.



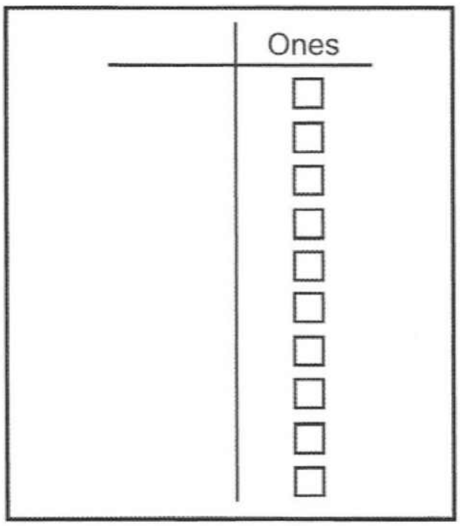
Repeat for all numerals up to nine, using the same dialogue. The child is told that numerals talk to him—they give him directives (not orders) for him to follow and tell him what he has to do.

His job is to know and respond to the language.

When you are sure the child understands the meaning of units, has facility with handling them, and can talk in terms of what he does with the numerals and rods, he is ready for the next step.

Teaching the meaning of "tens"

Using the preceding format draw ten blocks in the ones column.

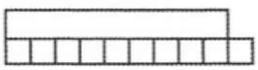


Teacher: It seems to me that as we use more and more rods they become more difficult to handle. Is there one large rod we can use in place of this number (10) of one rods? (Demonstrate how cumbersome it is to handle so many small rods.)

Child: This rod (as he picks up the "nine" rod).

Teacher: How will you prove to me that all of the one rods make the same size as that large rod?

Child: I'll measure them. *Child lines up the one rods next to or on top of the large rod.*



Teacher: What is the matter? How are they different?

Child: The big rod isn't big enough.

Teacher: What must you do?

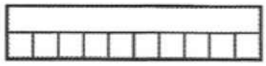
Child: Get a bigger rod.

Teacher: Which one will you use?

Child: The orange one (as he picks up the "ten" rod). .

Teacher: What will you do now?

Child: Measure them.



Teacher: Are they the same size?

Child: Yes.

Teacher: How many *one* rods do you need to make the same size as the orange rod? Count them.

Child: (After counting) ten.

Teacher: From now on instead of using so many *one* rods, can we use the orange one to show *ten* ones? Do you think it will be easier to handle than ten *ones*?

Child: Yes.

Teacher: Show me one *ten* rod.

Child: Picks up orange rod.

Teacher: Yes. Now how many *one* rods do we mean when we use the orange rod?

Child: Ten.

Teacher: Yes, we use an orange rod to mean ten *one* rods. Show me two *ten* rods. Show me three *ten* rods.

Child: (Responds accordingly.)

Teacher: Can we put the *ten* rod in the *ones* column?

Child: No.

Teacher: What is the reason for not being able to put the *ten* rod in the *ones* column?

Child: Because it is not a *one*.

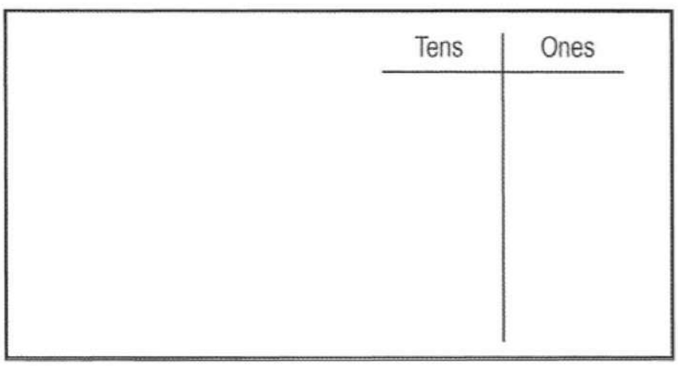
Teacher: Where should the *ten* rods go on your paper?

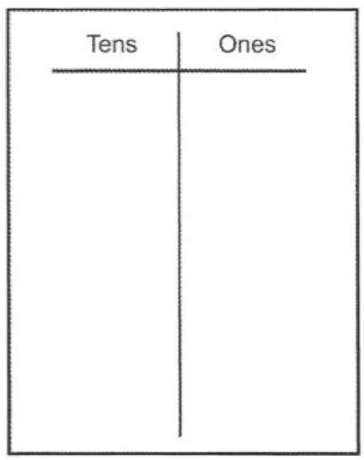
Child: In another column.

Teacher: Yes, what name do you think we should give the other column?

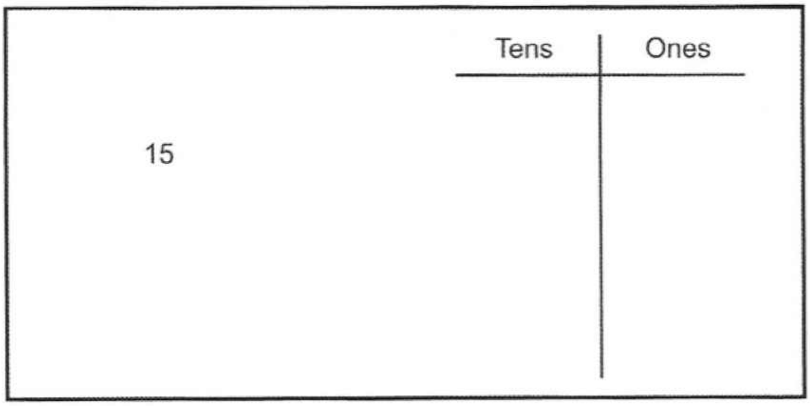
Child: *Tens*.

Teacher: Writes *tens* in proper column on chalkboard and on child's paper.





Teacher: The rule in our number system is: The greatest number in any column is nine. When we add one more, we must then move into the next column on the left. Now we will play with *tens* and *ones*. (The teacher writes 15 on the chalkboard and asks: "What is the name of this numeral?")



Child: Fifteen.

Teacher: What does this numeral mean?

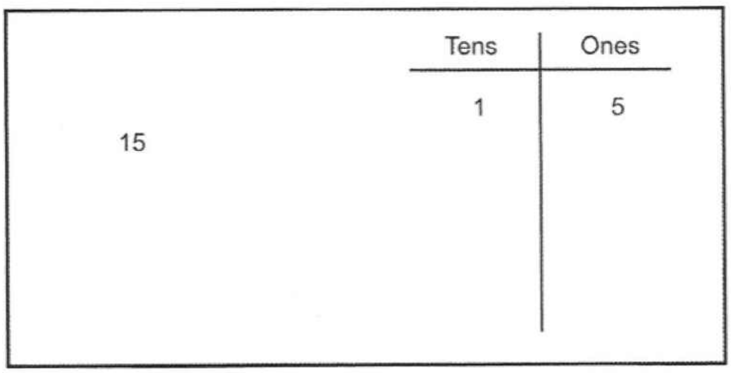
Child: Fifteen *ones*.

Teacher: What does it say?

Child: One *ten* and five *ones*.

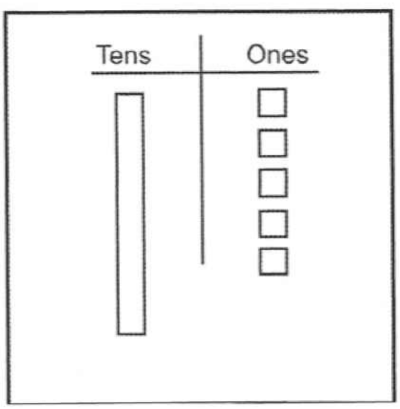
Teacher: Where do I write the one *ten*? The five *ones*?

Child: The *one* goes in the *tens* column and the *five* goes in the *ones* column.



Teacher: Using the rods, show me how this looks on your paper.

Child: (Selects one *ten* and five *one* rods and places them in the appropriate columns on his paper.)



Teacher: Tell me about what you have done.

Child: I have one *ten* and five *ones* which makes *fifteen*.

Repeat the same process going up to 99. Unfortunately, there is no rod which is equivalent to 100; therefore, arbitrarily select some object to represent 100: an eraser, a pencil, or an object different from the rods. Explain to the child that the reason you are using the 100 object is because it would be awkward to handle a rod 100 units long. Go through the same process discussing hundreds. Have the child make an additional column on his paper representing hundreds:

Hundreds	Tens	Ones

This same process can be carried on into thousands, ten thousands, or higher depending on how far you wish to go. I find that working into the hundreds is usually enough to teach children about our number system. Frequently, children become so enthusiastic with this approach that they want to go into the "bigger" numbers. They develop a feeling of importance and self-satisfaction as they learn to become conversant with numbers.

This understanding of our number system helps a child develop a working knowledge of numbers that he may apply to other processes of arithmetic. The purpose, in addition to understanding, is to help him develop ease of manipulation and a working relationship with numbers free from tension.

Meaning of Zero

How many people feel that zero means nothing? What would our number system be without the zero? How would we write numbers larger than nine? How would we designate a starting point in space?

It would be a good idea to have children work with a number system without the zero. They could use either our present number system or make one up. They would not use the zero when writing numbers which have the zero in them, such as 20, 306, or 509. They could do this in many ways; for example, they could write out the numbers: twenty, three hundred six, and five hundred nine. They could use a dash to illustrate the zero, such as 1- meaning ten, or 1-- meaning one hundred, or 3-6 meaning three hundred six. Having children work with a number system that does not contain the zero will give them insights into our number system and the need for the zero.

The zero is used in our number system as follows:

As a place holder:

When we have nothing in a column we use the zero to indicate that nothing is there. For example: the numeral *ten* is written as *one* and *zero* or *10*, which indicates that we have one *ten* and no *ones*. Three hundred and seven would be written as *three, zero, seven* or *307*, which indicates that there are three *hundreds*, no *tens*, and seven *ones*. When we see *07* on the calculator, this indicates that there are no *tens* and seven *ones*. We say that *zero* is a place holder because when there is nothing in the column, the *zero* holds the place of the column in our number system.

Activities to demonstrate the meaning of *zero* as a place holder are as follows:

Draw a numeral grid on the chalkboard as follows:

Tens	Ones

Teacher: (Verbally, give the child a number containing a zero; i.e., *ten*. Have the child tell you where to place the numerals representing ten.) Where should I place the numerals meaning *ten*?

Child: Write the *one* in the *tens* column.

Teacher: Write the numeral *one* in the *tens* column.

Tens	Ones
1	

Teacher: Are there any *ones*?

Child: No.

Teacher: Then, what should I place in the *ones* column?

Child: A *zero*.

Teacher: (Writes the *zero* in the *ones* column and asks, "What does that now tell us?")

Tens	Ones
1	0

Child: That tells us that there are no *ones*.

Teacher: (Pointing to what she has written.) What do these numerals say to us?

Child: The numerals tell us that there is one *ten* and no *ones*.

Teacher: When we speak about this, what do we say?

Child: We say *ten* but we could also say that there are ten *ones*. When we write it, we have to show one *ten* and no *ones*, otherwise we would have to show ten lines.

This same procedure is to be repeated using other multiples of ten, such as, twenty, thirty, forty, and up to ninety. The same thing is to be repeated for hundreds, from one hundred up to nine hundred. For hundreds draw the numeral grid as follows:

Hundreds	Tens	Ones

The dialogue is similar to that used when describing *tens* and is as follows:

Teacher: Where should I place the numerals meaning three hundred and seven?

Child: Put the *three* in the *hundreds* column, a *zero* in the *tens* column, and a *seven* in the *ones* column.

Teacher: (Writes the numbers in the appropriate column as directed by the child and asks, "What does this tell us?")

Hundreds	Tens	Ones
3	0	7

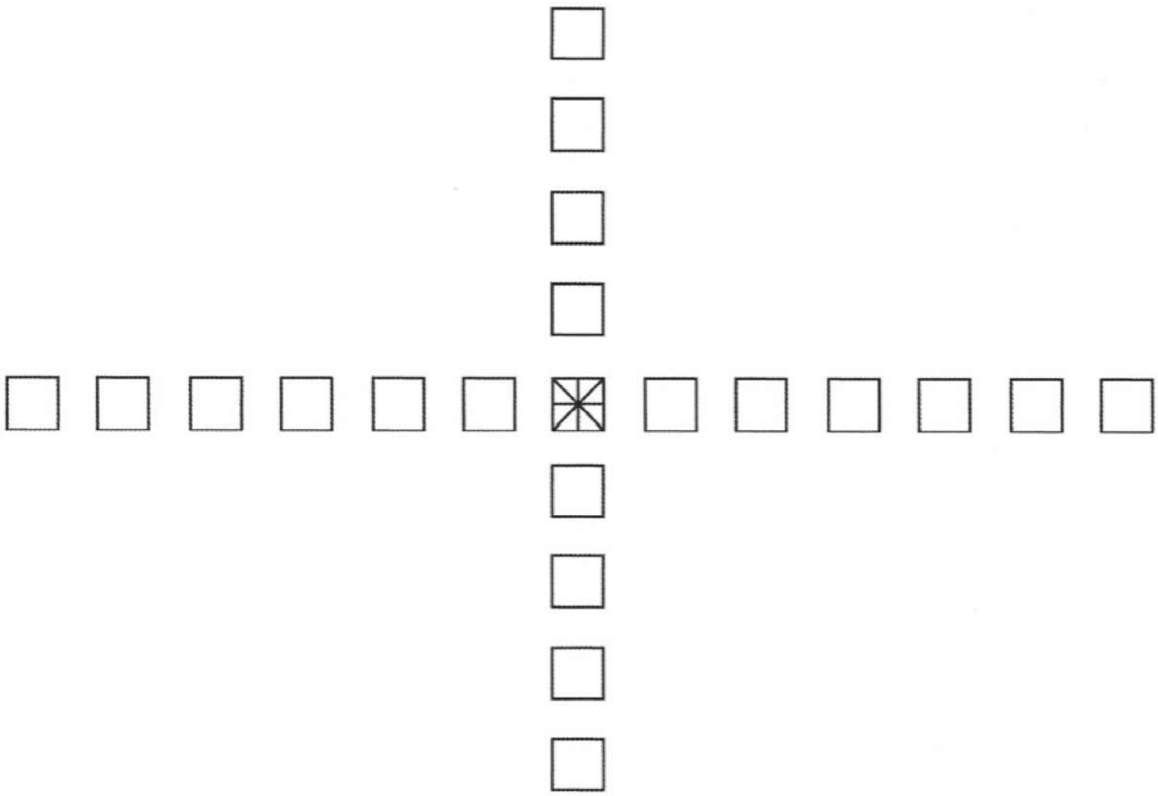
Child: This tells us that there are three *hundreds*, no *tens*, and seven *ones*.

Zero as a reference point or starting point.

Zero is a reference or starting point in space. We use *zero* to indicate where we start from. With a starting point it is possible to go above, below, right, left, or diagonally to that point. A person is the starting point for his world of space because all things exist in relationship to him. When working with altitude, sea level is the starting point, and we talk in terms of feet or miles above or below sea level. The *zero* enables us to have negative, as well as positive numbers which we note when using a thermometer.

Activities to demonstrate using *zero* as a reference or starting point are as follows:

(For additional activities refer to the game with the beanbag and soldiers in the section on Training Activities.) Have the children set up the tiles as described. The red square is the zero point and all of the other tiles exist in relationship to it, which will look as follows:



The child is instructed to knock over the soldier so many spaces to the right, left, above, or below the red square. Later this can be designated as plus or minus as follows: the tiles to the right and those above the red square are considered positive or plus; the tiles to the left and those below the red square are considered as negative or minus.

Were you to instruct the child to knock down the soldier two tiles to the right of the red square, you could say, "Knock down the soldier at plus two across."

You could teach the child the meaning of horizontal in place of across; vertical in place of up and down. Should you desire even more sophistication, you could label the horizontal as the "x" and the vertical as the "y." Your instruction for three to the left would become, "Knock down the solder at x-3." I would first use the terms up, down, and across, then the terms horizontal or vertical, and then the terms *x* or *y*. In this way the child can become aware of an evolving level of sophistication as a means of abbreviating language.

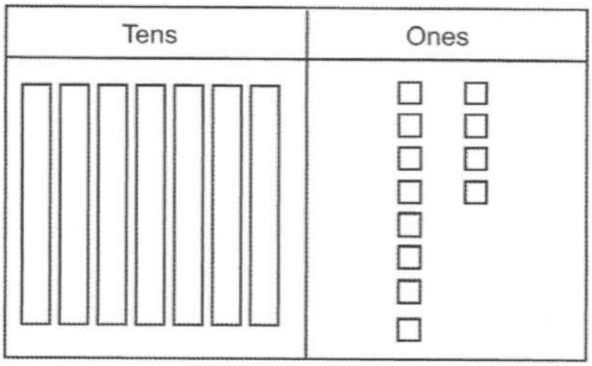
Stepping Stones

Another way of illustrating the *zero* as a starting point is to make a number line on the floor with blocks or colored tiles. At first, the tiles can be arranged either horizontally or vertically, then later, both horizontal and vertical.

Use a red or white tile as the ***zero*** point. Have the child start his walking at the ***zero*** point as follows:

1. Have him walk a certain number of steps to the right or left of the zero point.
2. After taking the steps he is to tell you that he is that number of steps to the right or left of zero.
3. As he develops proficiency have him use the terms *positive* and *negative* in place of *right* and *left.* You can then give him directions by saying, "Move *negative,* five steps," or "Move *positive* two steps."

Child:

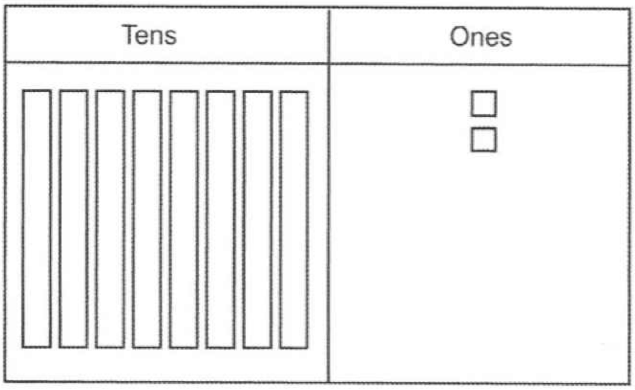


Teacher: What are you going to do now?

Child: Replace ten ones with one ten.

Teacher: Show me.

Child:



Teacher: Say what you see.

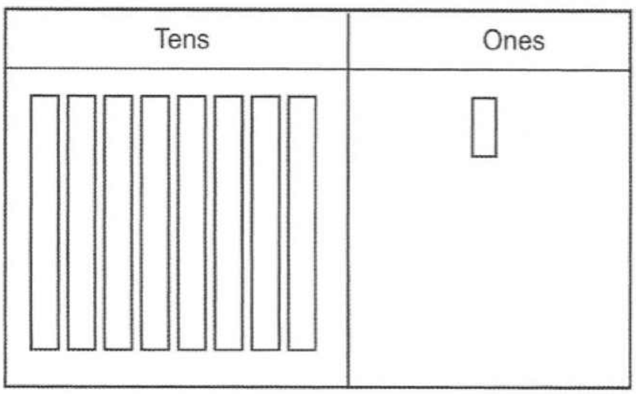
Child: Eight tens and two ones.

Teacher: What are you going to do now?

Child: Replace the two ones with a two block.

Teacher: Show me.

Child:



Teacher: Write it on the chalkboard.

Teacher: Write it on the chalkboard.

Child: (Writes as follows:)

	Tens	Ones
7 + 6 = ?	1	3

Teacher: What does that say?

Child: One ten and three ones.

Teacher: What is the name for one ten and three ones?

Child: Thirteen.

Teacher: Yes. Now answer this (pointing to the equation 7 + 6 = ?).

Child: Seven plus six equals thirteen.

Teacher: I agree.

Regrouping beyond the teens

The same procedure is carried out in the addition of numbers from twenty to ninety-nine.

Teacher: (Writes the following on the chalkboard.)

54 + 28 = ?

What does this say?

Child: Fifty-four plus twenty-eight equals what?

Teacher: In which column should I put the numerals?

Child: For the fifty-four, put the five in the tens column and the four in the ones column.

Teacher: (Writes on the chalkboard as follows:)

	Tens	Ones
54 + 28 = ?	5	4

Child: For the twenty eight, put the two in the tens column and the eight in the ones column.

Teacher: (Writes on the chalkboard, as follows:)

	Tens	Ones
54 + 28 = ?	5	4
	2	8

Teacher: Show me how this looks in terms of number.

Child: Put them in the ones column.

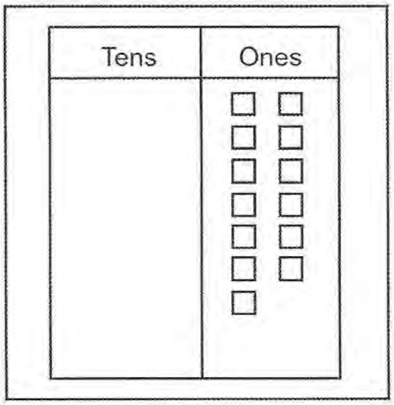
Teacher: (Writes on the chalkboard, as follows:)

7 + 6 = ?

Tens	Ones
	7 +6

Teacher: Show me how this looks in terms of number.

Child:

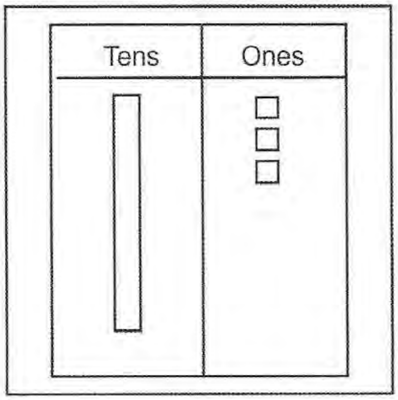


Teacher: What are you going to do now?

Child: Replace ten ones with one ten.

Teacher: Show me.

Child:



Teacher: Say what you see.

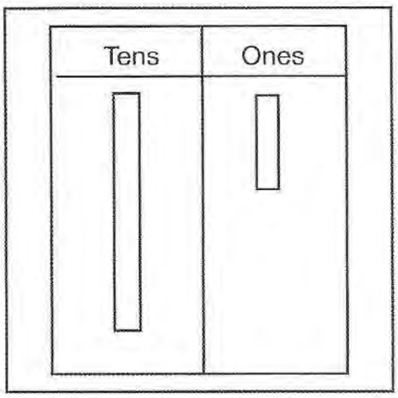
Child: One ten and three ones.

Teacher: What are you going to do now?

Child: Replace the three ones with a three block.

Teacher: Show me.

Child:



Teacher: How many blocks are in the ones column?

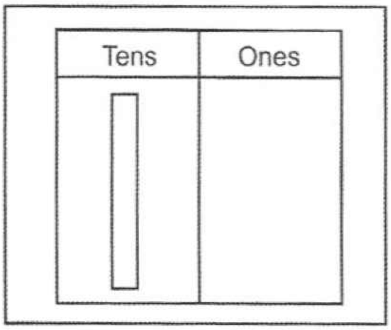
Child: (Counts the blocks and says, "Ten.")

Teacher: What are you going to do with the ten one blocks?

Child: Exchange it for a ten block. (Takes out a ten block and puts the one block in the box.)

Teacher: Where are you going to put the ten blocks?

Child: In the tens column.



Teacher: Tell me how you say what you see.

Child: One ten and no ones.

Teacher: Write it on the chalkboard.

Child: (Writes it as follows:)

Tens	Ones
1	0

Teacher: What does that say?

Child: One ten and no ones.

Teacher: What is the name of that symbol?

Child: Ten.

Teacher: Yes. Now answer this (pointing to the numeral sentence 9 + 1 = 10).

Child: Nine plus one equals ten.

Teacher: I agree.

Repeat the same procedure with all number combinations that equal a sum of ten. Give examples of adding two, three, and four numerals together to get a sum of ten.

Regrouping beyond ten

Teacher: (Writes the following on the chalkboard:)

$$\boxed{7 + 6 = ?}$$

What does this say?

Child: Seven plus six equals what.

Teacher: In which column should I put the numerals?

Teacher: On the chalkboard, write the answer to the question 5 + 2 = ?.

Child: (Writes the numeral 7 and says, "Five plus two equals seven.")

Teacher: I agree.

(Repeat the same procedure with all number combinations that equal a sum of nine or less. Give examples of adding two numerals together, such as 4 + 3 and three or four numerals together, such as 2 + 2 + 1 or 2 + 3 + 3 + 1 .)

Addition of two-digit numbers

Establishing the Regrouping to Ten

9 + 1 = ?

Teacher: (Writes the following on the chalkboard:)

What does this say?

Child: Nine plus one equals what.

Teacher: In which column should I put the nine?

Child: The ones column.

Teacher: (Writes on the chalkboard and asks, "In which column should I put the one?)

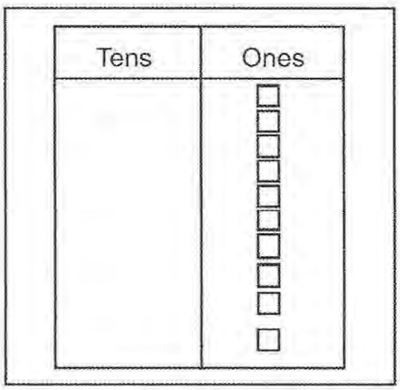
Child: The ones column.

Teacher: (Writes on the chalkboard, as follows:)

	Tens	Ones
9 + 1 = ?		9 +1

Teacher: Show me how this looks in terms of number.

Child:



Teacher: The rule in our number system is: The highest counting number in any column is how many?

Child: Nine.

Teacher: When one more is added to nine in a column, we must move into the next column on the what?

Child: Left.

What does this say?

Child: Five plus two equals what.

Teacher: In which column should I put the five?

Child: The ones column.

Teacher: (Writes on the chalkboard and asks, “In which column should I put the two?”)

5 + 2 = ?

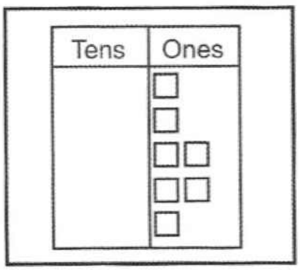
Tens	Ones
	5 +2

Child: The ones column.

Teacher: (Writes on the chalkboard, as follows):

Show me how this looks in terms of number.

Child:



Teacher: How many one blocks are there all together?

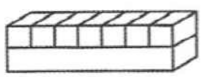
Child: Seven.

Teacher: Find the rod that is the same size as all of the one blocks together.

Child: (Picks out the seven rod.)

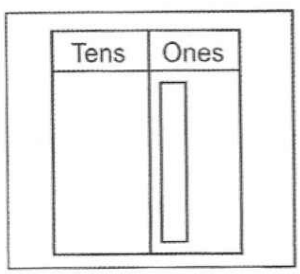
Teacher: Measure the seven rod by placing the one rods next to it.

Child:



Teacher: Put the seven rod in the ones column to take the place of the one rods, and put the one rods back in the box.

Child:



Teacher: Show me how you write the grouping of the blocks in order to take three steps when you start at the sixth step.

Child: (At the chalkboard, writes: "6 ⁄· 3 = 2.")

Teacher: Walk it out and show me.

Child: (Lays out one group of six blocks, divides them into three groups of two blocks, and walks it out.)

```
z x x x x x x
            6
z  x x  x x  x x
   2    2    2
```

Teacher: Tell me what it says.

Child: It says, take one group of six blocks and make it into three groups of two blocks each.

Teacher: I agree.

Repeat with other problems.

Using Cuisenaire Rods to Teach Mathematical Operations

Cuisenaire Rods are used to have the child go from the symbolic (numeral), to verbal, to concrete (number), to verbal, to numeral. This is done with the basic math operations as follows:

Addition

Set the child up with a set of Cuisenaire Rods and a piece of paper ruled off with the designation of our number system. On the chalkboard draw a designation of our number system.

Chalkboard

Tens	Ones

Child's Desk

Tens	Ones

The procedure used is similar to that discussed in the section, "Our Number System," on pages 91 to 97 and is done as follows:

Teacher: The rule in our number system is: The highest number of counting order in any column is nine (9). When one more is added to nine in a column, we must then move into the next column on the left.

Addition of single digit numerals

Teacher: (Writes the following on the chalkboard:)

$\boxed{5 + 2 = ?}$

Teacher: Tell me what it says.

Child: It says that two groups of four blocks each will give me one group of eight blocks.

Teacher: Walk it out and show me.

Child: (Groups the blocks into two groups of four, walks it out, puts them into one group of eight.)

```
Z X X X X  X X X X
        4       4
Z X X X X X X X X
                8
```

Teacher: I agree. (Repeat with other problems.)

Division:

a. The tiles are set up as in la above.

b. Have the child stand on the last tile (the "10" tile).

c. Develop the following dialogue:

Teacher: From where you are take five steps to zero; how many blocks will each step cover? (Or, if you step on every other block, how many steps will you have taken to get to zero?)

Child: I will step on every second block.

```
Z X X X X X X X X X X
                   10
                Z  X X  X X  X X  X X
                     2    2    2    2
```

Teacher: Show me.

Child: (Walks on the blocks.)

Teacher: I agree.

d. When the child gains proficiency in the step above, present the problem to him as follows:

Teacher: Divide nine steps by three; how many steps will you take and how will you group the blocks?

Child: I will take three steps and I will have three groups of three blocks each.

Teacher: Show me.

Child: (Takes three steps and makes three groups of three blocks each.)

```
Z X X X X X X X X X
                  9
Z  X X X  X X X  X X X
       3      3      3
```

Teacher: I agree.

e. When the child gains proficiency in the phase described above, work with the arithmetic denotation for division as follows:

Teacher: Show me how the statement, "Starting at the sixth step, how many blocks will each step cover if you take three steps to get back to zero?" will look when it is written on the chalkboard in arithmetic language.

Child: (Goes to the chalkboard and writes: "6 ÷ 3 = ?")

Multiplication:

a. The tiles are set up as in la above.

b. Have the child stand on the z tile.

c. Develop the following dialogue:

Teacher: Take three steps of two blocks each.

Child: (Takes three steps of two blocks each.)

```
z  xx  xx  xx
    2   2   2
```

Teacher: Group the blocks to show how you have put three steps of two blocks each into one group.

Child: (Does it.)

Teacher: I agree.

Teacher: How many blocks will you have moved through altogether?

Child: I have moved through six blocks altogether.

```
zxxxxxx
      6
```

Teacher: I agree.

d. When the child gains proficiency in walking by twos, threes and fives, present the problem to him as follows:

Teacher: Take four times two steps.

Child: (Takes four steps of two blocks each.)

```
z  xx  xx  xx  xx
                8
```

Teacher: Group the blocks to show how you have put four steps of two blocks each into one group.

Child: (Does it.)

Teacher: I agree.

Teacher: How many blocks will you have moved through altogether?

Child: I have moved through eight blocks altogether.

Teacher: I agree.

e. When the child gains proficiency in the phase described above, work with the arithmetic denotation for multiplication, as follows:

Teacher: Show me how the statement, "What is the total number of blocks you have moved through altogether when you take two steps of four blocks each?" looks on the chalkboard written in arithmetic language.

Child: (Goes to the chalkboard and writes: "2 x 4 = ?")

Teacher: Show me how you write the number of blocks you have moved through when you take two steps of four blocks each.

Child: (At the chalkboard writes: "2 x 4 = 8.")

Subtraction:

a. The tiles are set up in 1a above.

b. Have the child stand on the z tile.

c. Develop the following dialogue:

Teacher: Starting at the zero point, take six steps forward, turn around and then take three steps back towards the zero point.

Child: (Walks out the directions.)

$$\underline{3}\ \underline{2}\ \underline{1}$$
$$z\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ x\ x\ x$$
$$1\ 2\ 3\ 4\ 5\ 6$$

Teacher: From where you are, how many steps do you have to take to get back to the starting point?

Child: Three steps.

Teacher: I agree.

d. When the child gains proficiency in handling the mechanics of the activity of subtraction, the following dialogue may ensue:

Teacher: Starting at zero, take four steps forward, then two steps back toward zero.

(Before the child takes the steps, he is asked: "Where would you be when you are finished?)

Child: Two steps from zero.

Teacher: Show me.

Child: (Walks it out.)

$$\underline{2}\ \underline{1}$$
$$z\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ x\ x\ x\ x\ x$$

Teacher: Where are you now?

Child: Two steps from zero.

Teacher: I agree.

e. When the child gains proficiency in the phase described above, work with the arithmetic denotation for subtraction, as follows:

Teacher: Show me how the statement, "Take five steps forward, turn around and take four steps back towards the zero point," looks on the chalkboard written in arithmetic language.

Child: (Goes to the chalkboard and writes "5 - 4 = ?".

Teacher: Show me how you write the number of steps away from zero you will be when you have taken five steps forward and then four steps back.

Child: At the chalkboard, writes: "5 - 4 = 1 ."

Teacher: Walk it out and show me.

Child: (Walks it out.)

$$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$$
$$z\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ \underline{x}\ x\ x\ x\ x$$
$$1\ 2\ 3\ 4\ 5$$

Teacher: I agree. (Repeat with other problems.)

d. When the child gains proficiency in counting and relating his steps to zero, the following dialogue may ensue to develop number facts:

Teacher: Starting at zero, take three steps forward and then take six more steps forward.

(Before the child takes the steps, he is asked: "How many steps will you have taken altogether?")

Child: Nine steps altogether.

Teacher: I agree.

(Should the child be in error and state: "Ten steps altogether," the teacher is to say: "Show me." The child then walks it out. Should he still hold to "Ten steps," the teacher is to say: "I disagree," not "You are wrong!" The teacher then is to say: "Try it again and check it out; see if you can find your error.")

e. When the child gains proficiency in his number facts, the following dialogue to teach arithmetic denotation may ensue:

Teacher: Show me how the statement, "Take four steps forward and then take two more steps forward," looks in arithmetic language when it is written on the chalkboard."

Child: (Goes to the chalkboard and writes: 4 + 2 = ?)

Teacher: Show me how you write the number of steps you have taken altogether when you take four steps and then two more steps.

Child: (At the chalkboard, writes: 4 + 2 = 6.)

Teacher: Walk it out and show me.

Child: (Walks it out.)

$$\begin{array}{cccccccccc} z & \underline{x} & \underline{x} & \underline{x} & \underline{x} & \overline{x} & \overline{x} & x & x & x \\ & 1 & 2 & 3 & 4 & 1 & 2 & & & \end{array}$$

Teacher: I agree.

(Should the child be in error, handle it in the same way described above. Allow him to make the error on the chalkboard, walk it out, and discover his own error and the solution to the error.)

f. Present problems to the child requiring more complicated addition, such as the following:

Teacher: Writes the following type of problem on the chalkboard: 3 + 2 + 5 = ?

Child: Looks at the problem presented and makes a response.

Teacher: Walk it out and show me.

Child: (Walks it out and responds with the answer "Ten," and then goes to the chalkboard and writes the answer, "10.")

$$\begin{array}{ccccccccccc} z & \underline{x} & \underline{x} & \underline{x} & \overline{x} & \overline{x} & \underline{x} & \underline{x} & \underline{x} & \underline{x} & \underline{x} \\ & 1 & 2 & 3 & 1 & 2 & 1 & 2 & 3 & 4 & 5 \end{array}$$

Teacher: I agree. (Repeat with other problems.)

Using Stepping Stones to Practice Mathematic Processes

Stepping stones consist of a series of blocks which may be made of loose floor tiles or 8" pieces of 2 x 4 wood. They may be arranged in a variety of ways depending on the concept you wish to teach. They may be used as an additional aid to teach the following:

1. Zero as a starting or center point.
2. Arithmetic operations—addition, subtraction, multiplication, and division.
3. Positive and negative numbers.

The activities to illustrate these functions are as follows:

Zero as a starting point:

a. Select ten tiles and place them on the floor in a straight line. The first tile should be distinguished from the others by being a different color or with markings on it to make it different, as in the following:

```
z x x x x x x x x x
```

b. Have a child stand on the *z* tile.

c. Develop the following dialogue:

Teacher: Take three steps forward.

Child: (Takes three steps.)

```
z x x x x x x x x x
  1 2 3
```

Teacher: Where are you from your starting point?

Child: I am three steps from my starting point.

Teacher: We call the starting point, the zero point. How many steps are you from the zero point?

Child: I am three steps from the zero point.

d. Repeat the above having the child take 1, 2, 3 steps, etc., up to ten.

This type of activity helps the child relate number concept to a definite starting point in space, which is the zero. This concept is important to him for all future operations with numbers.

Addition:

a. The tiles are set up as in 1 a above.

b. Have the child stand on the z tile.

c. Develop the following dialogue:

Teacher: Starting at the zero point, take five steps forward and then take three more steps forward.

Child: (Walks out the directions.)

```
z x x x x x x x x x
  1 2 3 4 5 1 2 3
```

Teacher: How many steps have you taken altogether?

Child: Eight steps altogether.

Teacher: I agree.

1. Addition. Addition, the basis of all arithmetic processes, is a building up process, and the words associated with it always build or increase as follows:

   Add, plus, more, greater than, all together, sum, increase, build, larger, extend, enlarge, grow, magnify, make higher, gain, unite, join, combine,associate.

   The symbol for addition is: +.

2. *Subtraction.* Subtraction is a process of decreasing, and the words associated with it always decrease, as follows:

   Subtract, less, lessen, decrease, make smaller, remove, take away, take from, withdraw, left over, make lower, reduce, sever.

   The symbol for subtraction is: -.

3. *Multiplication.* Multiplication, the addition of like amounts, is a building up process where smaller groups of similar items are put into one large group, and the words used are as follows:

   Multiply, all together, times, product, enlarge, extend, grow, magnify, increase, build, unite, join, gather, combine, associate.

   The symbol for multiplication is: x; or . ; or ( ); or it is implied, as 2y.

4. *Division.* Division is a process where a large group is made into an equal number of smaller groups of equal content. It is the undoing of multiplication, and the words used are as follows:

   Divide, equal parts, amongst, each, part, separate, sever, distribute, asunder, goes into.

   The symbol for division is: ÷.

Working with arithmetic operations

The child in the classroom may be asked to make up problems using words to describe processes of arithmetic. After a child states his problem, other children might discuss the words used and the processes they imply. Examples of this are as follows:

1. John is playing with blocks. He has arranged them so that he has four piles of blocks with five in the first pile, six in the second, three in the third, two in the fourth. How many blocks does John have altogether? How many would be in each pile if John were to arrange them into four equal piles?

   The question, "How many blocks does John have altogether?" implies the arithmetic operation of addition.

   The question, "How many would be in each pile if John were to arrange them into four equal piles?" implies the arithmetic operation of division.

2. John has four equal piles of blocks arranged four to a pile. How many blocks would John have if they were combined into one large pile?

   The question, "How many blocks would John have if they were combined into one large pile?" implies the addition of like quantities, which is multiplication.

3. Bricks were ordered for a building. Four thousand three hundred bricks were ordered, but only four thousand bricks were actually used. How many bricks were left over?

   The question, "How many bricks were left over?" implies subtraction.

4. Twelve pieces of candy are to be shared equally among four boys. How many pieces will each boy receive?

   The question, "How many pieces will each boy receive?" and the word "equally" implies division.

4. Repeat the procedure using a vertical arrangement of the tiles and teach the child that *positive* is *above* and *negative* is *below.*

Thermometer

Use a thermometer to illustrate a vertical arrangement of numerals with a zero point. The use of positive and negative numbers relative to zero may be illustrated in this way.

Purpose

The purpose of working this through with the child (or with a group of children) is that you not only want them to write out what they are doing, you also want them to talk it out. This gives you an opportunity to hear what the child is thinking, and it gives the child an opportunity to express his thinking. Having the child express his thinking verbally helps him develop feedback, which in turn helps him develop an understanding of the process he is developing and the reason for what he is doing.

When the child makes an error, *do not* tell him he is wrong; rather tell him that you do not agree with him. Have him look at the situation and ask him to explain it to you. It is only by working with the process that the child will learn and have the experience become a part of him. When he is only concerned with being right or wrong to meet an adult's approval, his learning may not result in gaining insight into what he is doing. The adult may think the child has learned when he continually gets correct answers, but many times the child may fall down at the next level of performance, and no one can understand why.

Many parents report that their child used to do well in the lower grades, but as he went into the higher grades he did poorly. This may have happened because the child committed his work to memory without really understanding it. This failure might have been avoided by taking time at the beginning to make sure the child had insight rather than rote. When insight exists, a person can talk it out, and "owns" the information.

The Operations of Arithmetic

"Should I *add* or *subtract?*" "Am I supposed to *multiply* or *divide?*" These are the questions heard from children as they seek the correct operation to solve an arithmetic problem. Parents frequently report that their child does well in arithmetic computation, but when it comes to word problems he falls apart because he cannot determine which arithmetic operation is needed to solve the problem.

Never to be forgotten is the anguish of a young boy as he tried to work out an arithmetic problem and could not figure out the correct operation needed. He was not able to comprehend the language of the problem which would have told him the correct operation. All he could relate the written information to was the question, "What did the teacher say? Do I add or subtract?" Rather than learning the meaning of the language of the various operations of arithmetic, he had memorized rules. As he continued to search for the "right" operations, he became frustrated, angry, and depressed; finally coming to the conclusion that he was just plain stupid and could not learn. He viewed himself as a failure because other children not only seemed to do the work so easily, but actually seemed to enjoy doing it.

Continuous difficulty with arithmetic causes a child to develop fear and apprehension towards arithmetic and he protects his feelings by turning off. Turning off inner language starts a vicious cycle of failure, and *more* failure, with the result that the child's work suffers, and he finally avoids doing it. The need to help children develop freedom of function within themselves is again highlighted. Inner freedom and security with arithmetic is developed by the child through understanding and becoming facile with the language of arithmetic, the operations represented, and the words used to describe these operations. Children must know as many ways as possible to linguistically describe addition, subtraction, multiplication, and division.

Language of arithmetic operations

Following are some of the words which may be used to describe the various arithmetic processes:

Child: (Writes as follows:)

	Tens	Ones
54 + 28 = ?	8	2

Teacher: What does that say?

Child: Eight tens and two ones.

Teacher: What is the name for eight tens and two ones?

Child: Eighty-two.

Teacher: Yes. Now answer this (pointing to the numeral sentence 54 + 28 = ?).

Child: Fifty-four plus twenty-eight equals eighty-two.

Teacher: I agree.

Repeat the same procedure with number combinations that equal sums up to ninety-nine. This basic procedure can be carried into the hundreds or thousands using other concrete designations to represent hundreds or thousands.

Subtraction

As addition is a building process or a process of increasing, subtraction is a removal process or a process of decreasing. Knowing the number facts of addition will help a child with subtraction because addition verifies subtraction.

Subtraction of single-digit numbers

Teacher: (Writes the following on the chalkboard:)

What does this say?

6 – 3 = ?

Child: Six take away three equals what.

Teacher: In which column should I put the six?

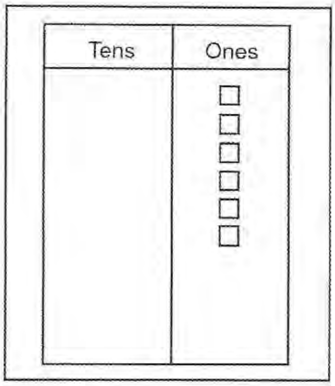
Child: The ones column.

Teacher: (Writes on the chalkboard, as follows:)

	Tens	Ones
6 – 3 = ?		6

Teacher: Show me how this looks in terms of number.

Child:



Teacher: Take away three one blocks; how many are left?

Child: (Removes three one blocks and says, "Three.")

Teacher: Write the following on the chalkboard:

$6 - 3 = ?$

Tens	Ones
	6
	− 3
	3

Teacher: On the chalkboard write the answer to the question 6 - 3 = ?

Child: (Writes the numeral 3 and says, "Six take away three equals three.")

Teacher: I agree.

Repeat the same procedure with all number combinations involving single digits.

Subtraction of two-digit numbers

a. Establishing the Regrouping of Ten

Teacher: (Writes the following on the chalkboard:)

$10 - 4 = ?$

What does this say?

Child: Ten take away four equals what.

Teacher: How should I represent this in terms of our number system?

Child: Write one in the tens column and zero in the ones column.

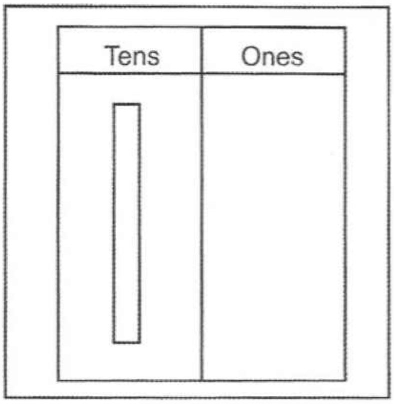
Teacher: (Writes on the chalkboard, as follows:)

$10 - 4 = ?$

Tens	Ones
1	0

Teacher: Show me how this looks in terms of number.

Child:



Teacher: Can you take away four ones from a solid ten block?

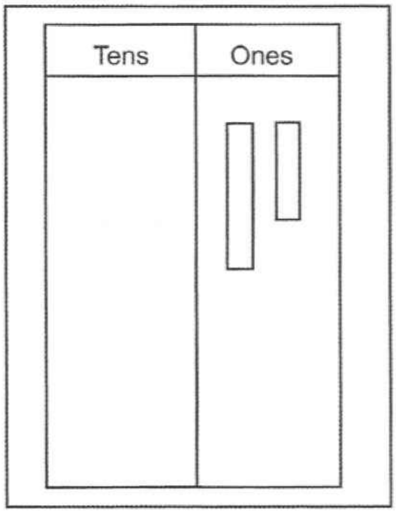
Child: No, you can't.

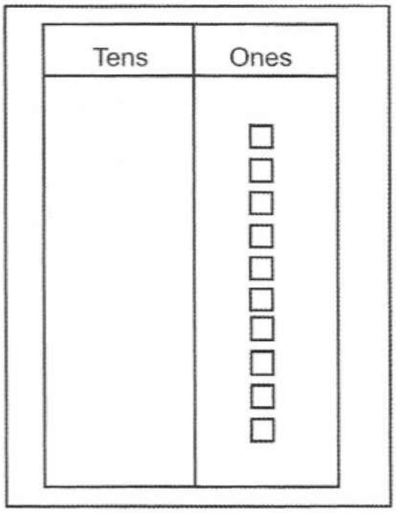
Teacher: What do you have to do?

Child: Cash in the ten block for ten ones or a six and a four block.

Teacher: Show me.

Child:

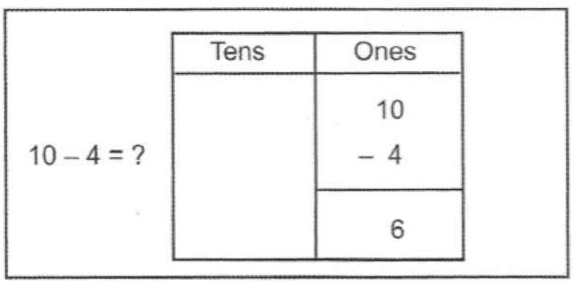




Teacher: Take away four, how many are left?

Child: (Removes four and says, "Six.")

Teacher: (Writes the following on the chalkboard:)



Teacher: On the chalkboard, write the answer to the question 10 - 4 = ?.

Child: (Writes the numeral 6 and says, "Ten take away four equals six.")

Teacher: I agree.

(Repeat the same procedure with all number combinations from 10-1 to 10-9.)

Using other two-digit numbers

Teacher: (Writes the following on the chalkboard:)

$\boxed{36 - 19 = ?}$

Teacher: What does this say?

Child: Thirty-six minus nineteen equals what.

Teacher: How should I represent this in terms of our number system?

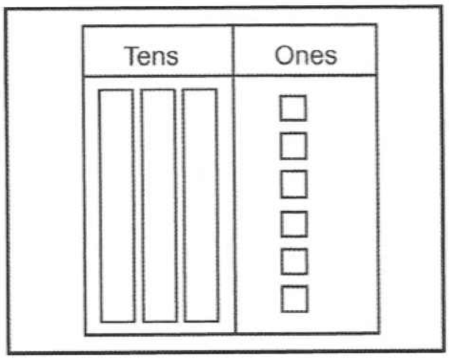
Child: Write three in the tens column and six in the ones column.

Teacher: (Writes on the chalkboard, as follows:)

Teacher: Show me how this looks in terms of number.

	Tens	Ones
36 – 19 = ?	3	6

Child:



Teacher: Can you take away nine ones from six ones?

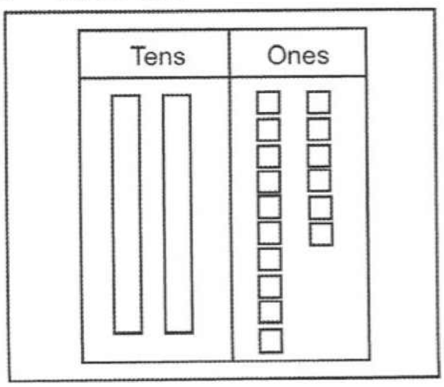
Child: No, I can't.

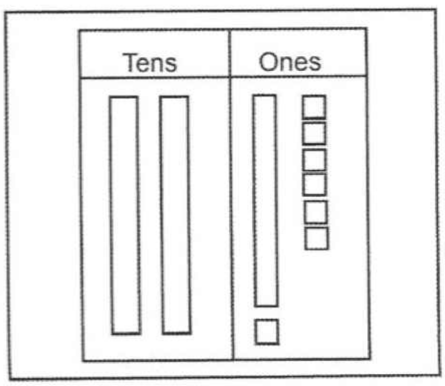
Teacher: What do you have to do?

Child: I have to cash in the ten block for either ten ones or a nine and a one block.

Teacher: Show me.

Child:





Teacher: Take away nine; how many are left?

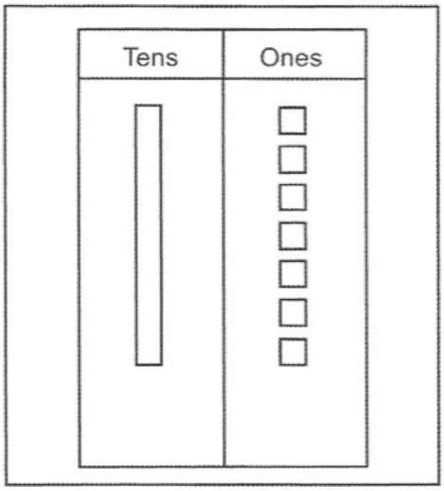
Child: (Removes nine ones and says, "Seven ones are left.")

Teacher: Can you take away one ten from two tens?

Child: Yes, that will leave one ten.

Teacher: Show me.

Child:



Teacher: (Writes the following on the chalkboard :)

36 – 19 = ?

Tens	Ones
2 ~~3~~ – 1	16 – 9
1	7

Teacher: On the chalkboard, write the answer to the question 36 - 19 = ?

Child: (Writes the numeral 17 and says, "Thirty six take away nineteen equals seventeen.")

Teacher: I agree.

Repeat the same procedure with other combinations of two-digit numbers.

Three digit (hundreds) numbers may be used with the same process.

Illustrate the relationship between addition and subtraction

To illustrate the relationship between addition and subtraction, do as follows:

Teacher: (Writes the following on the chalkboard :)

4 + 2 = ?

Teacher: What does this say?

Child: Four plus two equals what.

Teacher: How should I represent this in terms of our number system?

Child: Write four in the ones column, and under that write two in the ones column.

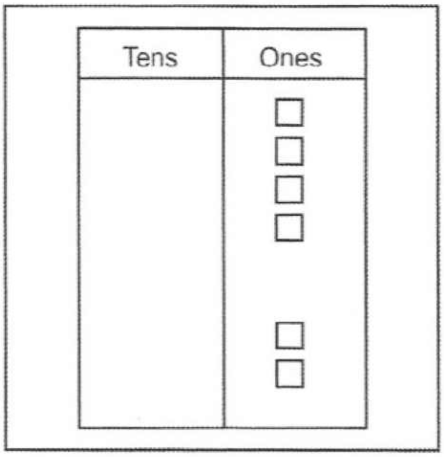
Teacher: (Draws the following on the chalkboard:)

4 + 2 = ?

Tens	Ones
	4 2

Teacher: Show me how this looks in terms of number.

Child:



Teacher: How many one blocks are there all together?

Child: Six.

Teacher: Remove two one blocks and how many will remain?

Teacher: Show me how this looks in arithmetic language.

Child: (Writes on chalkboard :)

$$6 - 2 = 4$$

Teacher: Replace the two one blocks and then remove four. How many will remain?

Child: Two.

Teacher: Show me how this looks in arithmetic language.

Child: (Writes on the chalkboard :)

$$6 - 4 = 2$$

Teacher: On the chalkboard, state in arithmetic language, the relationships between six, four, and two.

Child: (Writes on the chalkboard:)

$$\begin{gathered} 2 + 4 = 6 \\ 4 + 2 = 6 \\ 6 - 4 = 2 \\ 6 - 2 = 4 \end{gathered}$$

Teacher: Show this to me in terms of number.

Child: (Illustrates these functions using the Cuisenaire Rods as before.)

(Repeat using other number combinations until child demonstrates ease of handling this concept.)

Multiplication

When teaching multiplication, the following four concepts are to be conveyed to the child:

Multiplication is:

- The grouping of like amounts
- The addition of like amounts
- The counting by a given number
- The fact that when you learn half of the multiplication table, you know it all.

These four concepts are taught as follows:

Multiplication of single digit numbers

Teacher: (Writes the following on the chalkboard :)

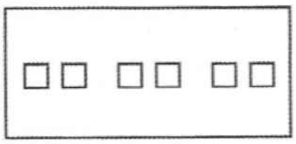
3 x 2 = ?

What does this say?

Child: Three times two equals what.

Teacher: In terms of the number system, this means that there are three groups of two, and when they are put together there will be one group of how many? Show me how this looks in terms of number.

Child: (Arranges the Cuisenaire Rods as follows at his desk.)



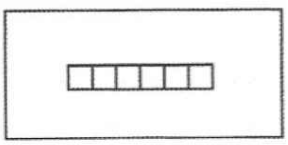
Teacher: Write on the chalkboard how this would look as an addition.

Child: (Writes the following on the chalkboard:)

2 + 2 + 2 = ?

Teacher: Put all the rods together and tell me how many there are when placed in one group.

Child: (Puts the rods altogether as follows:)



Three groups of two blocks makes one group of six blocks.

Teacher: I agree. On the chalkboard, write the answer to the question, "Three times two equals what?"

Child: (Writes the numeral 6 and says: "Three times two equals six.")

Teacher: Show me how this works when you count by two.

Child: (Points to 2 + 2 + 2 on the chalkboard, and as he points with his finger, says, "Two, four, six. Three twos equals six.")

Teacher: I agree. Now what does the following say? (On the chalkboard writes the following:)

$\boxed{2 \times 3 = ?}$

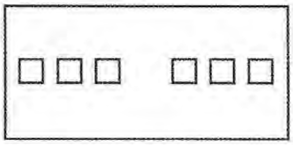
Child: Two times three equals what?

Teacher: How would you say this in terms of number?

Child: When two groups of three are put together, they will make one group of how many?

Teacher: Show me how this looks in terms of number.

Child: (Arranges the Cuisenaire Rods as follows:)



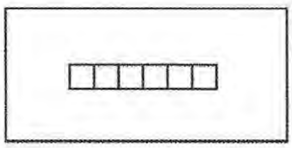
Teacher: Write on the chalkboard how this would look as an addition.

Child: (Writes the following on the chalkboard:)

$\boxed{3 + 3 = ?}$

Teacher: Put all the rods together and tell me how many there are when placed in one group.

Child: (Puts the rods all together as follows:)



Two groups of three blocks make one group of six blocks.

Teacher: I agree. On the chalkboard write the answer to the question, "Two times three equals what?"

Child: (Writes the numeral six and says, "Two times three equals six.")

Teacher: Show me how this works when you count by three.

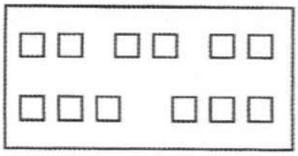
Child: (Points to a 3 + 3 on the chalkboard, and as he points with his finger, says, "Three, six. Two threes equal six.")

Teacher: How are three times two and two times three the same?

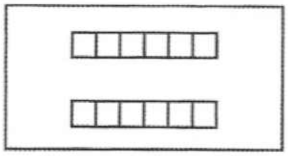
Child: They both equal one group of six.

Teacher: Show me by grouping the rods.

(Groups the rods as follows:)

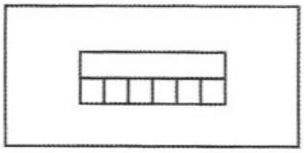


(Then puts each set together and gets the following:)



Teacher: Find the single rod that represents each of these groupings.

Child: (Selects the six rod and measures it with the group of six rods, as follows:)



Teacher: I agree.

Multiplication resulting in a product of two digits

Teacher: (Writes the following on the chalkboard :)

$6 \times 3 = ?$

Teacher: What does this say?

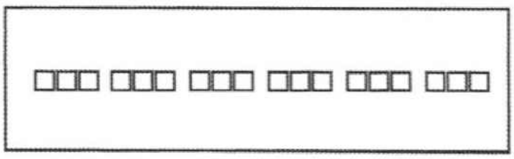
Child: Six times three equals what.

Teacher: How would you say this in words.

Child: When six groups of three are put together, they will make one group of how many? Or: A group of three taken six times equals how many?

Teacher: Show me how this looks in terms of number.

Child: (Arranges the Cuisenaire Rods as follows:)



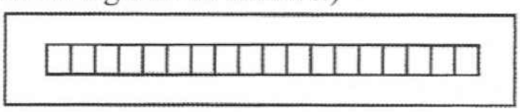
Teacher: Write on the chalkboard how this would look as an addition.

Child: (Writes the following on the chalkboard :)

$3 + 3 + 3 + 3 + 3 + 3$

Teacher: Put all the rods together and tell me how many there are when placed in one group.

Child: (Puts the rods all together as follows:)



Six groups of three blocks makes one group of eighteen blocks.

Teacher: I agree. On the chalkboard write the answer to the question, "Six times three equals what?"

Child: (Writes the numeral eighteen and says, "Six times three equals eighteen.")

Teacher: Show me how this works when you count by three.

Child: (Points to 3 + 3 + 3 + 3 + 3 + 3 on the chalkboard, and as he points with his finger says, "Three, six, nine, twelve, fifteen, eighteen. Six times three equals eighteen.")

Teacher: (Writes the following on the chalkboard:)

$$\boxed{3 \times 6 = ?}$$

What does this say?

Child: Three times six equals what.

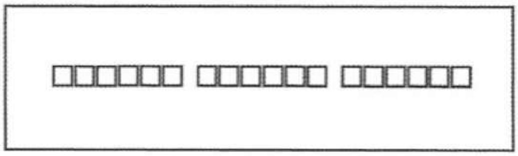
Teacher: How would you say this in words?

Child: When three groups of six are put together, they will make one group of how many?

Teacher: A group of six taken 3 times equals how many?

Teacher: Show me how this looks in terms of number.

Child: (Arranges the Cuisenaire Rods as follows:)



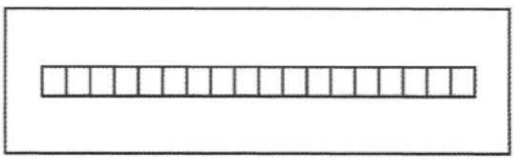
Teacher: Write on the chalkboard how this would look as an addition.

Child: (Writes the following on the chalkboard:)

$$\boxed{6 + 6 + 6}$$

Teacher: Put all the rods together and tell me how many there are when placed in one group.

Child: (Puts the rods all together as follows:)



Three groups of six blocks, makes one group of eighteen blocks.

Teacher: I agree. On the chalkboard write the answer to the question, "Three times six equals what?"

Child: (Writes the numeral eighteen and says: "Three times six equals eighteen.")

Teacher: Show me how this works when you count by six

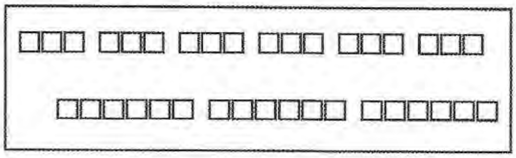
Child: (Points to 6 + 6 + 6 on the chalkboard, and as he points with his finger says, "Six, twelve, eighteen. Three times six equals eighteen.")

Teacher: How are six times three and three times six the same?

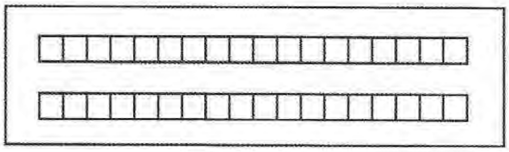
Child: They both equal one group of eighteen.

Teacher: Show me by grouping the rods.

Child: (Groups the rods as follows:)

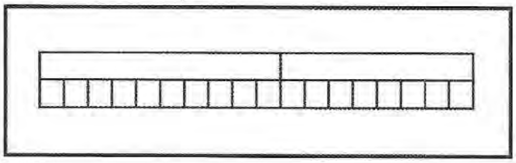


(Then puts each set together and gets the following:)



Teacher: Find the two rods that represent each of these groupings.

Child: (Selects the ten rod and the eight rod and measures them against the group of eighteen rods, as follows:)



Teacher: I agree.

Multiplication of two-digit numbers.

Teacher: (Writes the following on the chalkboard:)

$$15 \times 24 = ?$$

What does this say?

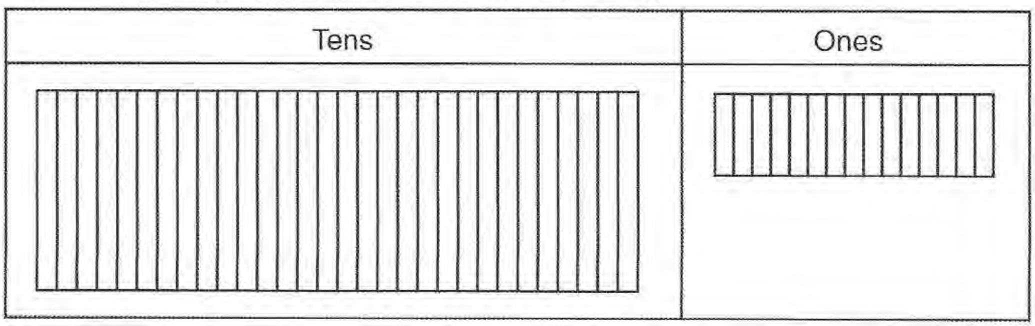
Child: Fifteen times twenty-four equals what.

Teacher: How would you say this in terms of number?

Child: When fifteen groups of twenty-four are put together they will make one group of how many? Or: A group of twenty-four taken fifteen times equals one group of how many?

Teacher: Show me how this looks in terms of number.

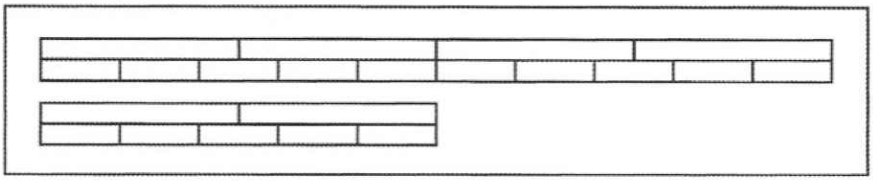
Child: (Arranges the Cuisenaire Rods as follows:)



There will be thirty tens and fifteen fours.

Teacher: Fifteen fours equals how many tens?

Child: (Lines up fifteen fours and matches it with ten blocks, as follows:)



Teacher: Replace the fifteen fours with six tens.

Child: (Removes fifteen fours and replaces them with six tens.)

Teacher: Add thirty tens to six tens and how large a group will you have?

Child: Thirty-six tens.

Teacher In our number system, what is the highest number you can have in a column before you change to another column?

Child: Nine.

Teacher: Show me how thirty-six tens looks according to our number system.

Child: (Draws the following on the chalkboard:)

Hundreds	Tens	Ones
3	6	0

Teacher: What does that say?
Child: Three hundreds, six tens, and no ones.

Teacher: What does that name?

Child: Three hundred sixty.

Teacher: I agree. Answer the question, "Fifteen times twenty-four equals what?"

Child: (Writes on the chalkboard:)

$$\boxed{15 \times 24 = 360}$$

Fifteen times twenty-four equals three hundred sixty.

Teacher: Iagree.Iwouldnowliketoshowhowthisworksusingnumeralsalone.(Writesonthechalk board :)

Five times four equals twenty ones and looks like this:

$$\boxed{5 \times 4 = 20}$$

Five times twenty equals one hundred ones and looks like this:

$$\boxed{5 \times 20 = 100}$$

Ten times twenty-four equals two hundred forty and looks like this:

$$\boxed{10 \times 24 = 240}$$

Add them all together and it looks like this:

$$\boxed{20 + 100 + 240 = 360}$$

The product of fifteen times twenty-four equals three hundred sixty.

(Continue with other similar problems using the same process.)

Developing the Multiplication Table.

On the chalkboard, draw the following:

Have the child work with you to fill in the various blanks as follows:

a. Line 1: Any number multiplied by one is the same number. This is known as "identity."

b. Line 2: Show that multiplying by two is the same as counting by two.

c. Line 3 to Line 10: Show that multiplying is like counting by that number.

d. Line 3: Show that knowing 2 x 3, you then know 3 x 2.

e. Line 4: Show that knowing 2 x 4 and 3 x 4, you then know 4 x 2 and 4 x 3.

f. Line 5: Show that knowing 2 x 5, 3 x 5, and 4 x 5, you then know 5 x 2, 5 x 3, and 5 x 4.

g. Line 6: Show that knowing 2 x 6, 3 x 6, 4 x 6, and 5 x 6, you then know 6 x 2, 6 x 3, 6 x 4, and 6 x 5.

h. Line 7: Show that knowing 2 x 7, 3 x 7,4 x 7, 5 x 7, and 6 x 7, you then know 7 x 2, 7 x 3 , 7 x 4 , 7 x 5, and 7 x 6.

i. Line 8: Show that knowing 2 x 8, 3 x 8, 4 x 8, 5 x 8, 6 x 8, and 7 x 8, you then know 8 x 2 , 8 x 3, 8 x 4 , 8 x 5 , 8 x 6 , and 8 x 7.

j. Line 9: Show that knowing 2 x 9, 3 x 9, 4 x 9, 5 x 9, 6 x 9, 7 x 9, and 8 x 9, you then know 9 x 2, 9 x 3, 9 x 4, 9 x 5, 9 x 6, 9 x 7, and 9 x 8.

k. Line 10: Show that knowing 2 x 10, 3 x 10, 4 x 10, 5 x 10, 6 x 10, 7 x 10, 8 x 10, and 9 x l0, you then know 10 x 2, 10 x 3, 10 x 4, 10 x 5, 10 x 6, 10 x 7, 10 x 8, and 10 x 9.

The completed chart looks like the one on the next page.

Do not preprint this chart-have the child work it out.

2	4	6	8	10	12	14	16	18	20
3		9	12	15	18	21	24	27	30
4			16	20	24	28	32	36	40
5				25	30	35	40	45	50
6					36	42	48	54	60
7						49	56	63	70
8							64	72	80
9								81	90
10									100

Division

The following concepts are to be conveyed to the child about division:

- Division is separating one large group of items into a certain number of equal groups.
- Division is the undoing of multiplication.

Division of single-digit numerals:

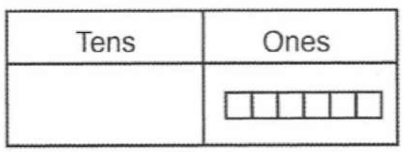
Teacher: (Writes the following on the chalkboard:)
What does this say?

$6 \div 2 = ?$

Child: Six divided by two equals what?

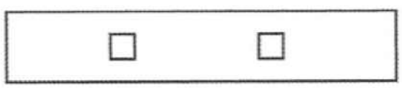
Teacher: In terms of number this means that one group of six when divided (or separated) into two equal groups will have how many in each group? Show me six ones.

Child: (Arranges the Cuisenaire Rods as follows:)



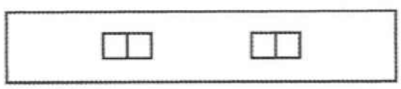
Teacher: Make two piles of one rod each.

Child: (Makes two piles of one each.)



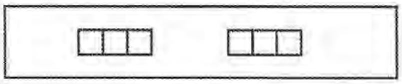
Teacher: Add one rod to each pile.

Child: (Adds one rod to each pile.)



Teacher: Add one more rod to each pile.

Child: (Adds one more rod to each pile.)



Teacher: Have you used up all of the one rods?

Child: Yes.

Teacher: How many rods are in each group?

Child: Three.

Teacher: Write the answer to the question on the chalkboard.

Child: (Goes to the chalkboard, writes the numeral 3, and says: "Six divided by two equals three.")

Teacher: I agree.

Demonstrating the relationship between division and multiplication:

Teacher: If you were to put two groups of three back into one group, how large a group would you have?

Child: Six.

Teacher: What did you do to arrive at that?

Child: I multiplied three times two.

Teacher: I agree. Show me how these relationships look in terms of numerals.

Child: (Writes the following on the chalkboard:)

$$\boxed{\begin{array}{c} 6 \div 2 = 3 \\ 3 \times 2 = 6 \end{array}}$$

Teacher: I agree. There is another relationship between six, three, and two. Six divided by three says what? (Writes on the chalkboard: $6 \div 3 = ?$)

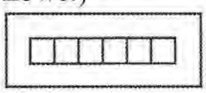
Child: Six divided by three equals what.

Teacher: How would you say this in terms of number?

Child: One group of six when divided into three equal groups will have how many in each group?

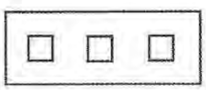
Teacher: Show me six ones.

Child: (Arranges the rods as follows:)



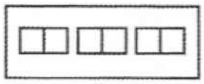
Teacher: Make three piles of one rod each.

Child: (Makes three piles of one rod each.)



Teacher: Add one rod to each pile.

Child: (Adds one rod to each pile.)



Teacher: Have you used up all of the one rods?

Child: Yes

Teacher: How many are in each group?

Child: Two.

Teacher: Write the answer to the question on the chalkboard.

Child: (Goes to the chalkboard, writes the numeral 2, and says: "Six divided by three equals two.")

Teacher: I agree. If you were to put two groups of three back into one group, how large a group would you have?

Child: Six.

Teacher: What did you do to arrive at that?

Child: I multiplied two times three.

Teacher: I agree. Show me how these relationships look in terms of numeral.

Child: (Writes the following on the chalkboard:)

$$6 \div 3 = 2$$
$$2 \times 3 = 6$$

Teacher: I agree. (Continues the same process using other single digit combinations that divide equally.)

Division of two-digit numerals

Teacher: (Writes the following on the chalkboard :)

$$96 \div 32 = ?$$

What does this say?

Child: Ninety-six divided by thirty-two equals what.

Teacher: How would you say this in terms of number?

Child: One group of ninety-six when arranged in thirty-two equal groups will have how many in each group?

Teacher: Show me ninety-six in terms of number.

Child: (Selects nine ten rods and six ones.)

Teacher: Can you separate them into thirty-two parts?

Child: No.

Teacher: What will you have to do?

Child: Cash the tens in for one rods.

Teacher: Show me ninety ones.

Child: Cashes in nine ten rods for ninety one rods.

Teacher: Make thirty-two piles of one rod each.

Child: (Makes thirty-two piles of one rod each.)

Teacher: Add one rod to each pile.

Child: (Adds one rod to each pile.)

Teacher: Add one more rod to each pile.

Child: (Adds one more rod to each pile.)

Teacher: Have you used up all of the one rods?

Child: Yes.

Teacher: How many one rods are in each pile?

Child: Three.

Teacher: Write the answer to the question on the chalkboard.

Child: (Goes to the chalkboard, writes the numeral 3, and says: "Ninety-six divided by thirty-two equals three.")

Teacher: I agree. Show me how this would look as a multiplication.

Child: (Writes the following on the chalkboard:)

$$\boxed{\begin{array}{r} 96 \div 32 = 3 \\ 3 \times 32 = 96 \end{array}}$$

Teacher: I agree.

Continue the same process using other two-digit combinations that divide equally.

Demonstrating a way to make it easier

Teacher: It seems that dividing by arranging the rods into piles to figure out an answer is a long and hard way. There must be an easier and quicker way. Does anyone have any ideas?

Have the children discuss possible ways to do it easier. Try whatever they say. After a reasonable period of time, if a child has not come up with an answer relating multiplication to the division, discuss division in relationship to multiplication, as follows:

Teacher: (Writes the following on the chalkboard :)

$$\boxed{15 \div 3 = ?}$$

What does this say?

Child: Fifteen divided by three equals what.

Teacher: How would you say this in terms of number?

Child: One group of fifteen when divided into three equal groups will have how many in each group?

Teacher: How would you state this in terms of multiplication?

Child: (Goes to the chalkboard, writes

$$\boxed{? \times 3 = 15}$$

and says: "What times three equals fifteen or three groups of how many in each group will make one group of fifteen?")

Teacher: I agree. How large will each group be?

Child: Five.

Teacher: Show me.

Child: (Arranges the one rods into three groups of five.)

Teacher: I agree. Write the answer on the chalkboard.

Child: (Goes to the chalkboard, writes the numeral 5, and says: "Fifteen divided by three equals five.")

Teacher: I agree.

Continue the same process using other combinations that divide equally.

Division of numerals resulting in a remainder.

Teacher: (Writes the following on the chalkboard:)

$$\boxed{17 \div 4 = ?}$$

What does this say?

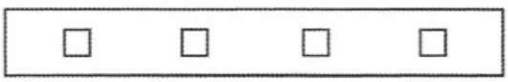
Child: Seventeen divided by four equals what.

Teacher: How would you say this in terms of number?

Child: One group of seventeen when divided into four equal groups will have how many in each group?

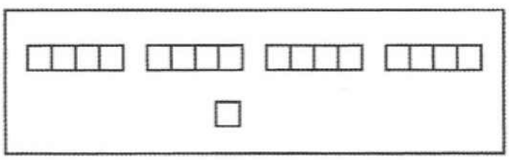
Teacher: Make four piles of one rod each.

Child: (Makes four piles of one rod each.)



Teacher: Add one rod to each pile until you use up all of the rods.

Child: (Adds one rod to each pile until he notices he has one rod left over.)



Teacher: Were you able to use up all of the rods equally?

Child: No, I have one rod left over.

Teacher: How would you write what you have in terms of numeral?

Child: I would write four with one left over.

Teacher: Show me.

Child: (Goes to the chalkboard, writes the 4 remainder of 1, and says: "Seventeen divided by four equals four with a remainder of one.)

$17 \div 4 = 4$ remainder of 1

Teacher: I agree. Show this to me as a multiplication.

Child: (Writes the following on the chalkboard:

Stated as a multiplication would look like this:

$(? \text{ X } 4) + 1 = 17$

$(4 \times 4) + 1 = 17$

Four times four equals sixteen plus the remainder of one equals seventeen.

Teacher: I agree.

Continue the same process using other combinations that do not divide equally.

Fractions and decimals

Fractions and decimals are beyond the scope of this chapter; however, using the Cuisenaire handbook and some creativity, a teacher can apply the principles of language to these functions.

Working with Money

When the concept of money is first introduced in a classroom, the children already know something about it because they have had a great deal of exposure to money. Many children have handled money before they understood what money was all about. Some children put pennies in a piggy bank when they are quite young. They hear about money in the home whenever they ask for something new, such as a toy, a game, a bicycle, ice cream, clothing or something else they may have a whim for. Usually they hear their parents telling them that their daddy doesn't have enough money. There are times when the child is sent to the grocery store, drug store, or candy store to buy something for his mother or father. He is given money and he knows that he is expected to bring home money in the form of what his parents call "change." Money is something children will have to live with all of their lives.

There are many activities the classroom teacher can do to give money more meaning and understanding. Some activities are as follows:

1. Have the children set up a model store in the classroom. This may be done as follows:
   a. The children discuss and decide what items to have in their classroom store. They may have pencils, erasers, paper, notebooks, toys, milk, Kleenex, and other items that they may want.
   b. Do not make money available to the children.
   c. Have them discuss how to procure items from the classroom store.
   d. Lead them into the concept of trading or bartering something that they already have for something that they want. As they trade back and forth they will realize that sometimes they have to make a decision between keeping something that they really like and want for something that they want even more. When this happens, ask them if they can think of a way that they can have the item they want to purchase and still keep the item they already have.
   e. Lead the children into an understanding that the only way they can keep objects they want is through the medium of money. Make up gold colored items which in fantasy can be considered as being precious because there is not too much of it. This will serve as a precious and rare metal, which

in reality can be compared to gold. Have hiding places around the room where you put this rare metal (similar to a mine). Have some of the children become prospectors and hunt the gold (this can be tied in with the gold rush of 1859 and other historical incidents which concern man's hunt for this precious metal). Distribute some of the mock-gold to the children in equal quantities and use this as a medium of exchange—commodities for gold (this can tie into a lesson on weights as well, but keep it simple). Show how some people will become gold exchangers or break the gold down into smaller amounts, while others may accumulate more. A whole lesson in economics can be taught around this concept by having some children do things for other children to earn money.

(You might discuss the feelings of the children who suddenly become affluent when they discover gold. Discuss the feelings of the children who have limited resources. This can help lead to sociological understanding.)

f. After the children see how they exchange something valuable for commodities, then substitute the gold for pennies. (If mothers wish, they can make up small bags for the pennies, or teach the children how to make a small money pouch.) Have the children work with large quantities of pennies.

g. Teach convenience in handling by having them realize how cumbersome, awkward, and time consuming the handling of pennies becomes. Discuss this inconvenience with them and elicit from the children that it would be much easier to have a way of representing a larger number of pennies with coins or paper of greater denomination. It is important to have them realize that whatever the larger unit will be, it must relate to the penny or one-cent piece.

   (1) When discussing coins, it may be more meaningful to go into the historical background of the various types of coins. Possibly, one of the children or a parent has a coin collection and might be willing to share the experience of seeing old coins.

   (2) Show a relationship between the penny or one cent piece and the single Cuisenaire rod.

      (a) Demonstrate to the child how a nickel means the same and takes the place of five pennies, just as the yellow rod (5-unit rod) means the same and takes the place of five one rods.

      (b) A dime means the same and takes the place of ten pennies, just as the orange rod (10-unit rod) means the same and takes the place of ten one rods.

      (c) A quarter means the same and takes the place of twenty-five pennies.

   (3) Ask the children which is easier to carry around-five pennies or a nickel, ten pennies or a dime, etc.

h. Show the children how coins can become heavy as you start getting into larger quantities. Discuss the need for lighter money in larger quantities, such as paper money. Use a dollar bill to show the following relationships:

100 pennies	
20 nickels	
10 dimes	Each is the same as a dollar bill.
4 quarters	

i. Have the children make change from the money that they are using. Suggest to parents that they take their children shopping and discuss various prices with them. Have them talk about the relative costs of items, especially food, where there is such a large variety of prices. Have them watch the checker in the supermarket and allow them the privilege of paying for the food and counting the change. When they get home have the children check the cash register tape. (Seeing the cash register tape and price designations in stores will make the child aware that money is written in two ways in terms of the symbols "$" and "¢" and in terms of decimals. When they see an item in the supermarket marked, they will note that it will usually be in terms of $, yet the cash register tape will show it marked in terms of decimals. Explain the following:

(1)Money written as 4¢, 2094¢, or 49¢ is being related to pennies.

(2)Money written as $.04, $.20 or $.49 is being related to parts of a dollar of 100 pennies (4/100, 20/100, or 49/100), which means four parts of a dollar, twenty parts of a dollar, or forty-nine parts of a dollar; which then relates back to one hundred pennies.)

2. Have the children do a lesson in history or social studies by tracing the evolution of money and its need.
   a. Start with the barter system and then show the need for a medium of exchange as we know it today.
   b. Explain letter of credit, money, checks and credit cards.
3. Have the children use games involving the use of money, such as Monopoly.
4. Have the children look through magazines and cut out ads where money is designated.

   Discuss this with the children.

Summary

When mathematics is treated as a language used to symbolically describe concrete relationships, a child will be able to learn the relationships between number and numeral. Talking the language of mathematics will enable him to handle the operations of mathematics, which are so essential to daily living and his future in the educational process. The initial time it takes to develop this process will pay dividends in future performance and remove the need for remedial work. As a remedial tool, it will open the child to learning.